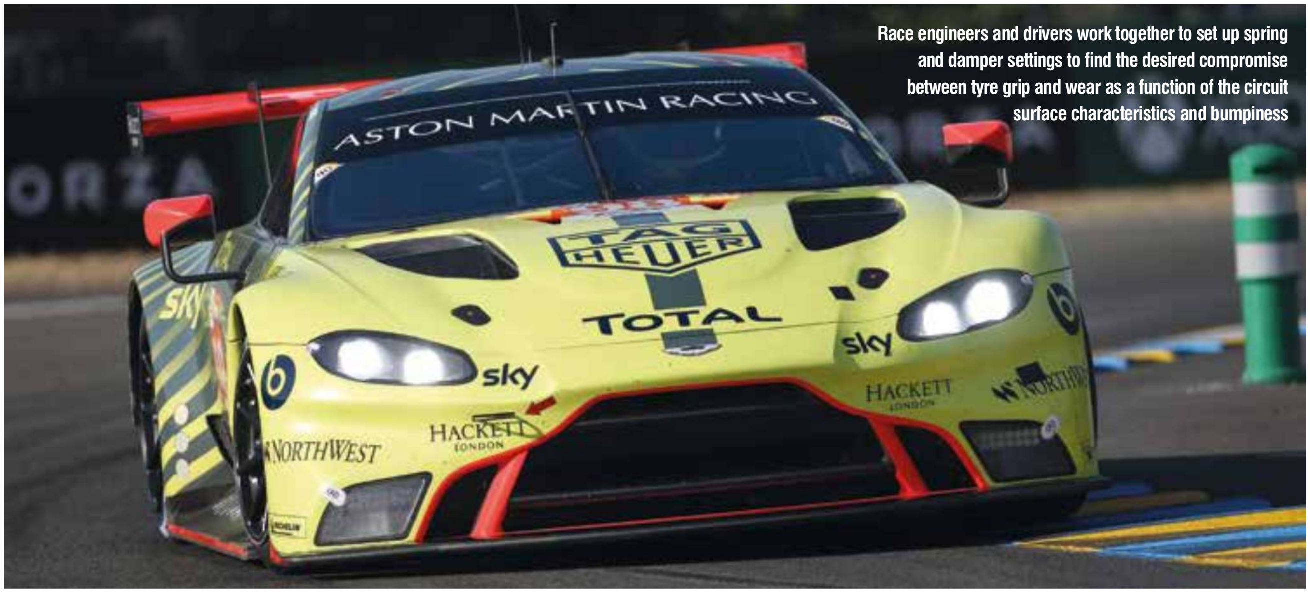
Of springs and dampers

The meaning and usefulness of suspended mass natural frequency numbers

By CLAUDE ROUELLE



Race engineers and drivers work together to set up spring and damper settings to find the desired compromise between tyre grip and wear as a function of the circuit surface characteristics and bumpiness

In the last article we stated that spring stiffness choice is based on experience and subjective appreciations, as much as it is on objective science. The determination of the spring stiffness is dependent of the track macro (bumps, kerbs, slope, banking) and micro (asphalt roughness) definitions, the track temperature, the car mass, inertia and aerodynamic properties, the tyre (in itself a dark science) grip, wear and thermal characteristics, the effects that suspension stiffness can have on the car reliability and, finally, driver skill.

Most of these inputs are not easily qualifiable and quantifiable. However, there are two fundamental engineering tools that we can bring into our decision process.

Shapes of things

The first one is the definition of track bumpiness. What are the shapes of the track bumps and how often, and at what speed, do the tyres hit these bumps? This will help in the spring stiffness choice. We will discuss this further in a future article on a quarter car model simulation. That simulation tool allows us to define spring and damper depending what the race engineer and driver are looking for, and the compromise they will accept between tyre grip and wear, ride height consistency and its effect on aerodynamic performance and expected response.

The second one is simply a determination of the suspended mass natural (undamped) frequency. The basic formula for this is

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{M}}$$

where,

K is the wheel rate expressed in N/m
M is the suspended mass expressed in kg
f is the suspended mass frequency and is expressed in cycles per second or Hertz (Hz)

Put simply, the wheel rate is a 'virtual' spring that represents the effect the spring has at the liaison of the wheel with the ground. Wheel rate will be in series with the tyre rate (or tyre stiffness if you prefer).

The wheel rate, K, is the spring rate divided by the square of the motion ratio.

$$K = Wheel\ rate = \frac{Spring\ rate}{MR^2}$$

The motion ratio is the ratio between wheel movement and spring movement, as explained in more detail in the box below.

Some people have difficulty understanding the reason for the use of the square of the motion ratio to calculate wheel rate from spring stiffness. Here is a simple explanation. If you take the principle of conservation of energy you can write that

$$(Force\ to\ move\ the\ wheel\ up) * wheel\ travel = (Force\ to\ move\ the\ spring) * spring\ movement$$

That equation supposes there isn't any friction in the rod ends and / or the bellcrank bearings. These frictions always exist in practice but, for the simplification of the demonstration, we will consider them negligible.

As the spring force is nothing other than the spring stiffness * , the spring movement in the previous equation can be re-written as

$$(Wheel\ rate * wheel\ travel) * wheel\ travel = (Spring\ rate * spring\ movement) * spring\ movement$$

Or

$$Wheel\ rate = \frac{Spring\ rate}{MR^2}$$

For those still not convinced, **Figure 1** shows a simple sketch extracted from the OptimumG Advanced Vehicle Dynamics seminar that will help full comprehension.

On the left, the real spring, and on the right, the 'virtual' spring that represents wheel rate. In this example, the motion ratio is two, that is to say the wheel moves twice as much as the spring. If wheel movement is double the spring movement, it is as if the 'virtual' spring (wheel rate) is half the spring stiffness. However, as you can see, the force is also half of the force on the spring. So, if we have twice the movement and half of the force, we have a wheel rate that is a quarter of the real spring stiffness. As the motion ratio is two, the wheel rate is the spring rate / 2^2. Hence the use of the *square* of the motion ratio.

Note that the notion of the motion ratio is applicable to the damper too, as illustrated by the sketch in **Figure 2**.

$$Damping@wheel = \frac{Damping\ @damper}{MR^2}$$

Here are a few useful comments and warnings to avoid often made mistakes:

- Spring stiffness is expressed in Newtons per metre (N/m) not in Newtons per millimeter (N/mm). That is a classic error.
- Be aware this equation is for one corner of the car only. Third (heave) springs are not taken into consideration here, nor are anti-roll bars.
- Look at the main formula and keep things in practical perspective. Because the natural spring stiffness is a function of the square root of the spring stiffness, when you double the spring stiffness, you only increase frequency by 41 per cent, because the square root of two is 1.41.
- Do not forget that wheel rate is in series with the tyres (and possibly some badly designed suspension compliance). Springs in series are always softer than their stiffer spring, and you do not double the whole suspension stiffness when you double the spring.

  When your car is not working and you suspect the suspension is too stiff, *or* too soft, changing the spring stiffness by only five per cent will not make a big difference.
- Motion ratio is the ratio between wheel movement and spring movement, not the other way around. We must be careful here as some engineers in specific racing series (NASCAR especially) often express motion ratio as spring movement vs wheel movement. That is not right or wrong, we just need to make sure everybody uses the same definitions in their calculations.

  Be careful, though, because 1.1² = 1.21. An error of 10 per cent on the motion ratio (because of non-respect of suspension parts manufacturing tolerance, for example) is an error of 21 per cent on wheel rate.
- On a suspension such as McPherson, motion ratio will always be bigger than one, and the wheel will always be moving more than the spring.
- On a car with a push or pull rod and rocker (some call it a bellcrank) suspension, spring movement can be bigger that wheel movement. The motion ratio in this case will be less than one.
- In most racing suspensions, the spring and damper are assembled in one unit, often called a coilover. The NASCAR front suspension, where spring and damper are not on the same axis, is an exception.
- A motion ratio smaller than one (where the spring and damper move more than the wheel) will generate more spring and damper movement than the wheel, and therefore more spring and damper speed and acceleration.

  For a given wheel up and down movement at a given frequency, we will get the same frequency at the spring and the damper (again, supposing compliance and friction of the rod ends and rocker bearings are negligible).The same frequency but a longer stroke results in higher damper speed and acceleration, whereas with higher damper speed we have more resolution and adjustability.

  To use a very basic example, it will be easier to control wheel movement with a damper that has a speed in a -250 to +250mm/sec window than one with a -50 to +50mm/sec window.

Figure 1: Motion ratio between wheel rate and spring rate

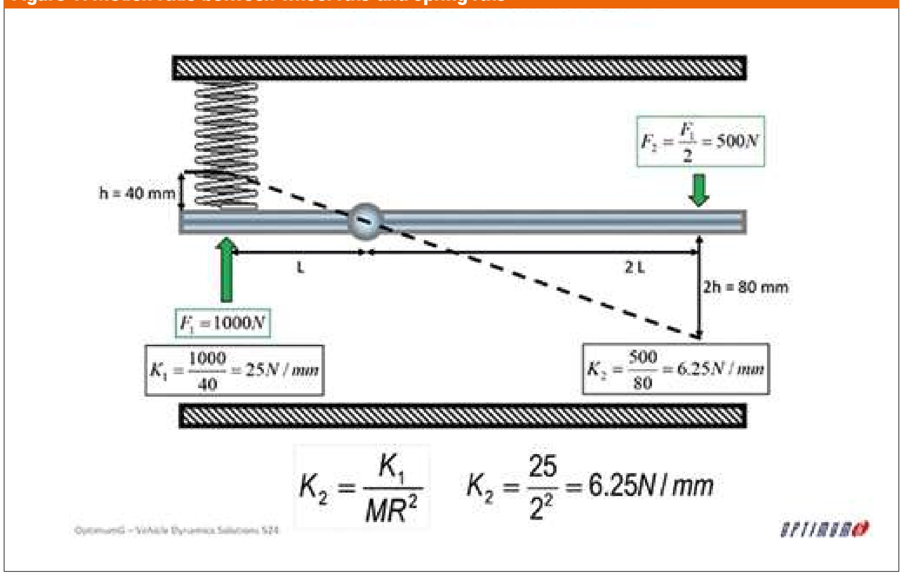
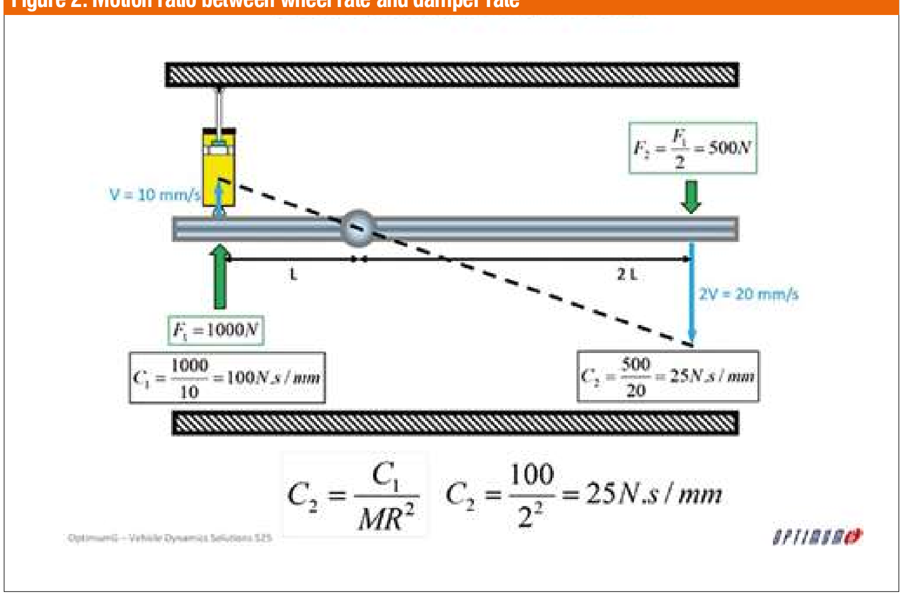


Figure 2: Motion ratio between wheel rate and damper rate



The advantage of the suspended mass natural frequency is it links both masses, kinematics of the motion ratio and suspension stiffness

- At the other end, bigger speed and bigger speed changes (accelerations) create the risk of damper cavitation. Risk of cavitation in a damper can be decreased with higher gas pressure, but at the cost of higher friction.
- As damper force (N) = damping rate (N/(mm/sec)) * damper speed (mm/sec), the same damping force can be achieved with less damping rate and more damping speed (that is achieved with a small motion ratio) than with more damping rate and less damping speed (that is achieved with a high motion ratio).
- For a given wheel movement and given wheel force, a small motion ratio will result in a longer and softer spring and a longer and softer damper, but keep in mind the implications damper and spring sizes and characteristics will have on weight and packaging.

  Small motion ratios seem appealing for their higher speeds and window of adjustments, but impose serious restrictions on the car designer in terms of packaging. Weight and packaging are always bigger challenges in cars like Formula 1 than, say, Baja cars.
- Motion ratio, MR, is not necessarily – in fact rarely – a constant number. By design and / or because of compliance, it is often a function more than a number.
- In fact, a non-linear motion ratio could have some advantage, especially for cars with aerodynamic ground effect. With these, we want a relatively soft suspension at low speed to get maximum tyre grip (we have explained in previous articles how the tyre 'hates' vertical load variation) and a stiff suspension at high speed to maintain a low ride height, despite the downforce that is square of the speed sensitive. There are three ways to achieve this compromise, and they can be used separately or together: variable spring rate (achieved with changing the spring pitch, wire diameter and spring outside diameter along its length), variable motion ratio and bump stops (made of polyurethane or even assemblies of Belleville washers).

  For example, if motion ratio is 0.9 at high ride height (low speed) and 0.8 at low ride height (high speed) with a spring constant stiffness of 400N/mm, wheel rate will vary from 400 / 0.92, which is 494N/mm to 400 / 0.82, which in turn is 625N/mm. That is a 25.6 per cent wheel raising rate. Not negligible, and not something to be ignored.

Spring stiffnesses themselves are rarely constant. If you put a spring on a spring tester, measure the stroke and the force, and make a graph of force vs movement you will rarely get a straight line. Without going into the details of the spring stiffness equation, we will simply remind you that when the spring is compressed, there will be more and more contiguous coils and the spring stiffness increases when the number of active coils decreases. From this point of view, torsion bars offer a more constant stiffness than a regular spring.

Practical constraints

At the end of the day, besides the equations developed here, the choice of motion ratio is most often dictated by practical weight and packaging constraints, and the answer to this simple question: do you design a coilover unit and decide its dimensions and characteristics to adapt it to a given existing suspension, or do you design a suspension around a given damper?

In other words, a Formula 1 car designer imposing damper and spring sizes and characteristics to a subcontractor is a very different story to a Formula Student team that was given a free set of dampers via some sponsorship, and has to design its car's suspensions around the imposed dampers.

Taking the following equations together,

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{M}}$$

$$K = Wheel\ rate = \frac{Spring\ rate}{MR^2}$$

we can define the spring stiffness for a given frequency and a given motion ratio.

$$Ks = 4\,\pi^2 f^2\, M\, MR^2$$

Where *Ks* is the spring stiffness (N/m), *M* the suspended mass (kg), *f* the suspended mass natural frequency (Hz) and *MR* the motion ratio (non-dimensional).

For example, let's consider a car's left front corner mass (measured on scales) that is 250kg, with a non-suspended mass of 42kg. The suspended mass will therefore be 208kg. If the motion ratio is 0.95 and the targeted natural frequency is 4Hz, the spring will be $Ks = 4\,\pi^2\ 4^2\ 208\ 0.95^2$ = 118,545N/m or 118.5N/mm, or 675lbs/in for our old fashion American friends.

The experience has shown that natural suspended mass frequencies (also called undamped frequencies) are in the following regions: for passenger cars – 1.1Hz (very soft) to 1.8Hz (very stiff); for non-aerodynamic racecars – 1.5-3.0Hz, and for aerodynamic racecars – 3.0-7.0Hz.

Yes, 7Hz seems particularly high, but let's not forget this kind of situation occurs on front suspension of aerodynamic racecars for which front ride height control is critical, and such cars are often seen with an additional third (heave) spring, and most probably the use of bump stops, too.

What's important to note is that each car mass and suspension are different. Each track or rally special stage is different. How then do we compare the suspension stiffness of a heavy racecar with a light rally car? The advantage of the suspended mass natural frequency is it links both masses, kinematics of the motion ratio and suspension stiffness, so it is a common denominator that allows us to compare car suspension stiffness.

But how stiff is too stiff? How soft is too soft? Now we are back to the intuitive and experimental approach describe in our previous article. At least, the suspended mass natural frequency is giving us the beginning of a reference point on the map of development. Or what we call a 'magic number'.

So, when your car works well, be sure to note the front and rear natural frequency. Then, the day you are lost, at least you can go back to this reference.

Slip Angle is a summary of Claude Rouelle's OptimumG seminars, which are held worldwide throughout the year. The Data Driven Performance Engineering seminar presents a number of data acquisition and analysis techniques that can be used by engineers when making decisions on how to improve vehicle performance.

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CONTACT
Claude Rouelle
Phone: + 1 303 752 1562
Enquiries: engineering@optimumg.com
Website: www.optimumg.com