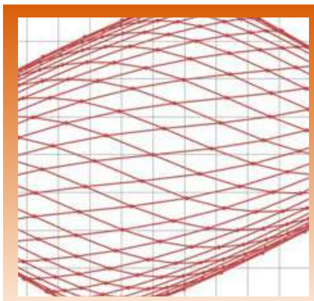

Slip Angle provides a summary of OptimumG's seminars

How to calculate a corner's importance

In Part 2 of OptimumG's explanation of track asymmetry Claude Rouelle looks at the impact of individual corners on lap time

In last month's article we explained how to make use of a GG diagram as a way to understand the race track. By calculating the percentage of distance the car spends in a given area of the GG diagram, we were able to objectively quantify the asymmetry of a track. Based on the amount of time the car spends in each sector of the diagram, we can get a very good estimate of how much distance the car spends in braking, accelerating, cornering and combined situations.

Even though the GG diagram can give us an idea of the asymmetry of the track, it is not able to show us the relative importance of each corner. A slow corner exit followed by a long straight, for example, could have more influence than a high speed corner. When our goal is to quantify the importance of each corner on lap time, we need to make use of simulation. We can, for example, artificially increase the mechanical grip of the tyres by five per cent only in a specific corner of the track. Once we do that for all the corners and check the gains in lap time, we are able to rank them in importance.

Figure 1 shows the lap time gain for each corner at Monza. The y-axis represents the percentage of lap time gain. We can observe, by looking at **Figure 1**, the influence of each corner in the overall lap time. Turn 3 and Turn 5 are the least sensitive to an increment in grip, while Turn 4 and Turn 11 are the most sensitive.

Now if we sum all the lap time gains for left and right turns, we will have, respectively, a total lap time gain of 0.38 per cent and 0.53 per cent, which indicates that we may benefit from having an asymmetric set-up in our racecar here.

Since the GG diagram only considers the resultant vehicle acceleration, it doesn't give us an indication of how much each tyre is being used. However, if we make use of a reliable tyre model and a good simulation tool, we can calculate the sliding energy that is being applied on each tyre, giving us an estimation of tyre usage.

When our goal is to quantify the influence of each corner at the track on the car's lap time we need to make use of simulation

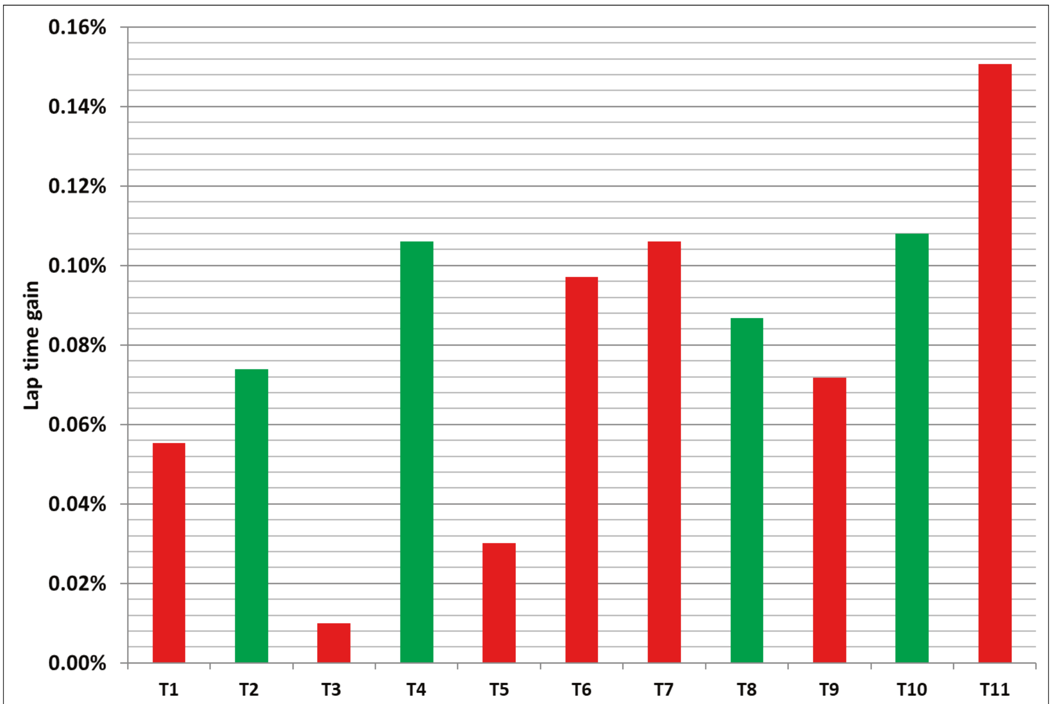

Figure 1: Simulation of a five per cent grip increase *vs* the lap time gain. Red columns are right turns and green are left

Sliding tyre energy

To calculate the sliding power on the tyre, we first need to compute the slip speeds, both in the longitudinal and lateral directions, as described in **Equations 1** and **2** in **Table 1**.

The resultant slip speed is then defined as the vector sum of the longitudinal and lateral slip speeds **(Equation 3)**. The second step is to calculate the resultant tyre force, which is the vector sum of the lateral and longitudinal forces **(Equation 4)**. Finally, the combined sliding power is calculated as the product between the combined tyre forces and the combined slip speed **(Equation 7)**.

If we then integrate the combined tyre sliding power over a full lap, we then end up with the sliding energy of that tyre **(Equation 10)**.

In the equations **V** is the translational speed of the centre of the wheel, **α** is slip angle, **Ω** is angular speed, **SR** is the tyre slip ratio, F_x and F_y are, respectively, the longitudinal and lateral forces in the tyre coordinate system. Linear speeds are given in metres per second, angular speeds in radians per second, angles in radians, forces in Newton, power (**P**) in Watt and energy (**E**) in Joules.

By using the same approach used in the previous article we can divide sliding energy into nine sections: pure acceleration; combined acceleration out of a right turn; pure right cornering; trail braking going into a right corner; pure braking; trail braking going into a left corner; pure left cornering; combined acceleration out of a left turn; and centre of the GG diagram. We can then plot all sections and compare for each track how the tyre energy is distributed.

Figure 2 shows an example of this type of analysis, made for different race tracks.

We can see here how the energy of the rear right tyre varies from track to track. At Spa, tyre energy is concentrated in pure left cornering and combined acceleration out of a right turn condition, while at Silverstone it is distributed between pure right cornering, pure left cornering and at centre. We can also see that at Imola, the only counter-clockwise circuit, tyres spend more energy in right combined acceleration. The different energy distributions shown here can help us make decisions about the amount of damping (compression damping has a huge influence on tyre temperature), toe, camber, cross weight, aerodynamic balance, brake balance, traction control, and differential settings that we will use for each of the circuits.

Total tyre energy

Another effective way of evaluating race track asymmetry is by looking at the difference between left and right tyre energy at both front and rear axles. Left-to-right asymmetry tends to be larger at more asymmetric race tracks. **Figure 3** shows the total energy spent on each tyre, for different race circuits.

We can clearly see that, at Le Mans, front and rear left wheels use more energy than front and rear right wheels. At Imola, the front and rear right tyres are being used more.

Total tyre energy, however, doesn't give you a fair comparison between tracks with different lengths. For example, comparing the total energy at Le Mans, which is 14km long, with Imola, which is 5km long. Therefore, when comparing different race tracks, we need to divide the total tyre energy by the track length, and analyse the tyre energy per kilometre, as shown in **Figure 4**.

We can see that, although, Le Mans is the longest circuit it is the one which uses less tyre energy per kilometre when compared with the other tracks. Paul Ricard and Silverstone have a big difference in the energy used in the front tyres.

Table 1: Tyre energy equations

Equation 1 – Lateral slip speed	$v_{lateral} = V * \sin(\square)$
Equation 2 – Longitudinal slip speed	$v_{longitudinal} = V * \cos(\square) * SR$
Equation 3 – Resultant slip speed	$v_{resultant} = \sqrt{v_{lateral}^2 + v_{longitudinal}^2}$
Equation 4 – Resultant tire force	$F_{resultant} = \sqrt{F_X^2 + F_Y^2}$
Equation 5 – Longitudinal sliding power	$P_x = F_X * v_{longitudinal}$
Equation 6 – Lateral sliding power	$P_y = F_Y * v_{lateral}$
Equation 7 – Combined tire sliding power	$P = F_{combined} * v_{combined}$
Equation 8 – Longitudinal sliding energy	$E_x = \int P_x$
Equation 9 – Lateral sliding energy	$E_y = \int P_y$
Equation 10 – Resultant sliding energy	$E = \int P$

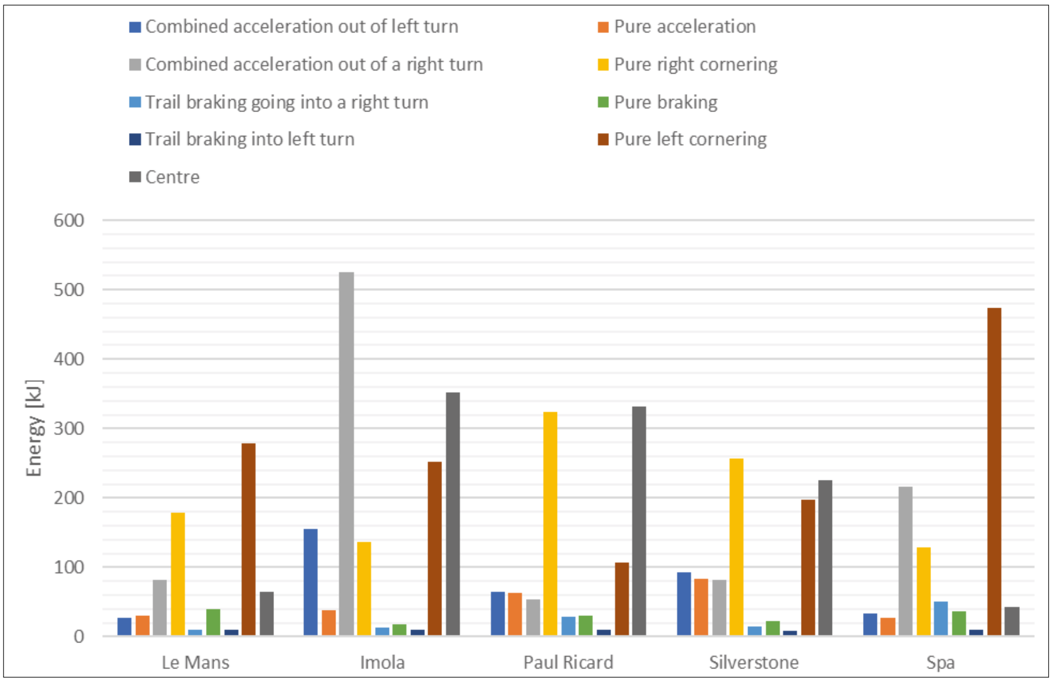

Figure 2: Total energy distribution of rear right tyre during a lap at different tracks. Note pure left cornering at Spa

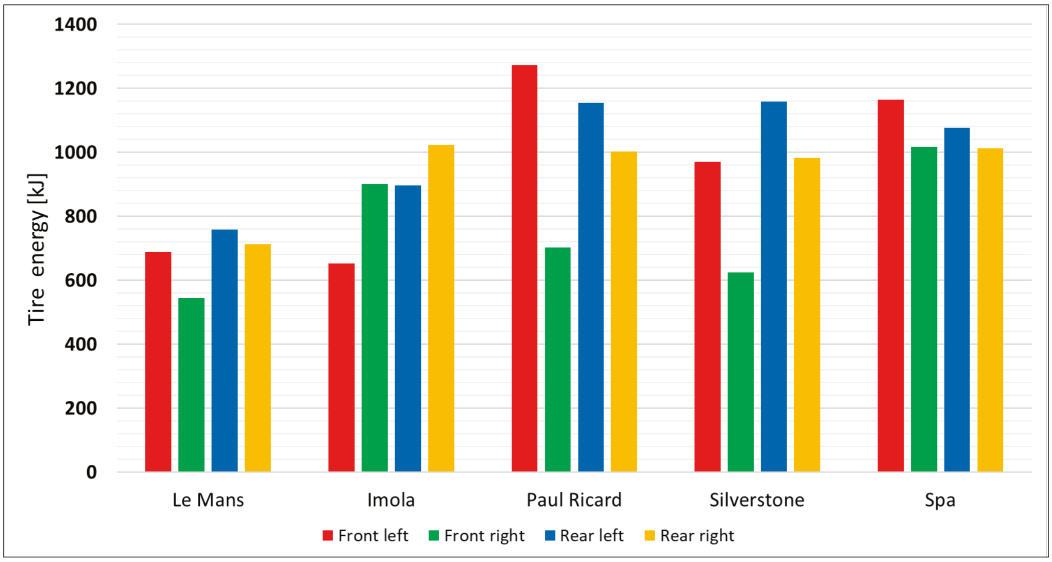

Figure 3: Total tyre energy in each wheel at different tracks. Note that at Imola front and rear right tyres are used more

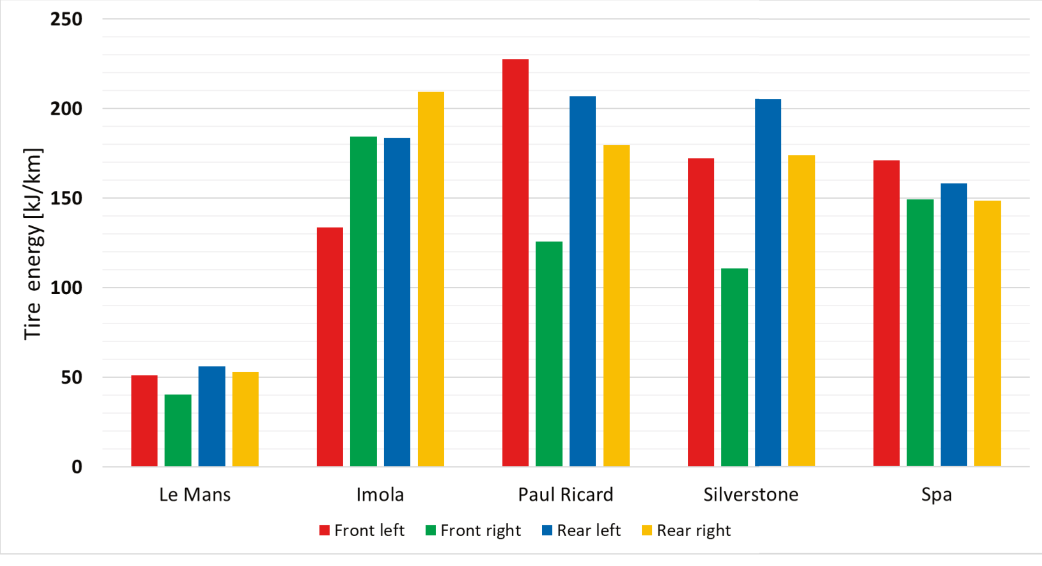

Figure 4: The tyre energy per km at different tracks – this is required as these race circuits are all of different lengths

Table 2: Front/rear and left/right energy calculation

$$Front/rear\ Energy\ Distribution = \frac{(FL\ Energy + FR\ Energy)}{(FL\ Energy + FR\ Energy + RL\ Energy + RR\ Energy)}$$

$$Left/right\ Energy\ Distribution = \frac{(FL\ Energy + RL\ Energy)}{(FL\ Energy + FR\ Energy + RL\ Energy + RR\ Energy)}$$

FL = front left, etc.

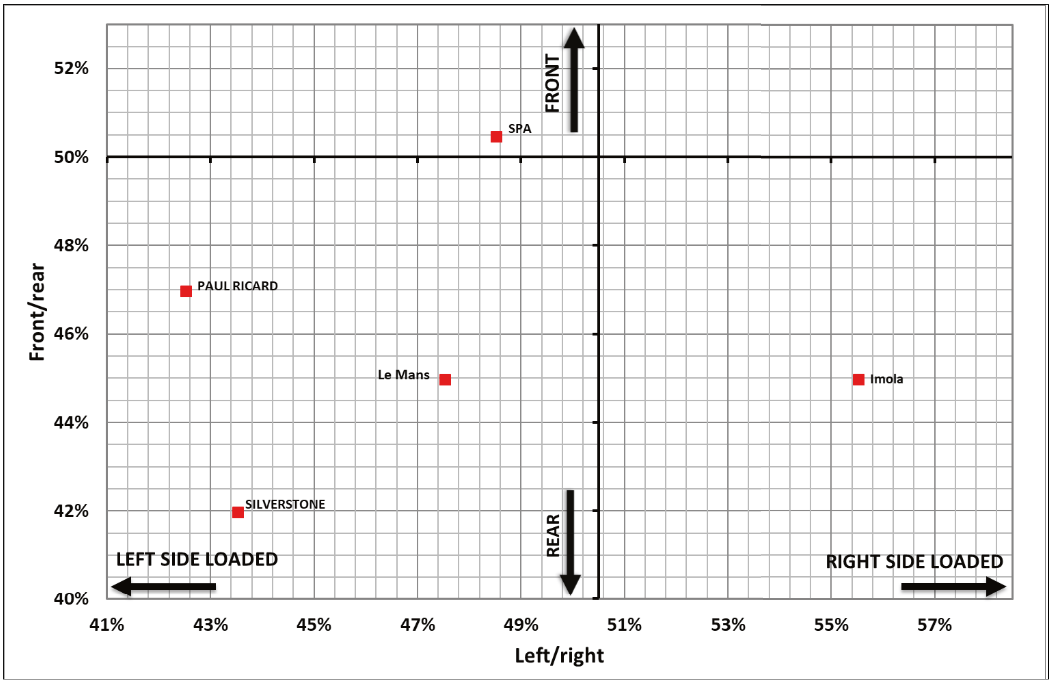

Figure 5: This gives us a visual reference as to how the sliding energy is distributed between tyres at different tracks

Although Le Mans is the longest circuit, it is the one which uses less tyre energy per kilometre when compared with the other race tracks

Spa actually has an even energy distribution between all the tyres, suggesting a symmetrical track.

Another method of comparing different race tracks is by calculating the front-to-rear and left-to-right tyre energy distributions, as shown in **Table 2**. Once we calculate these values for different circuits, we can plot them in a chart, as shown in **Figure 5**. This chart gives us a visual reference of how the sliding energy is being distributed between the tyres at different race tracks.

Conclusion

Even though the GG diagram we explored last month is useful, we can only draw a few conclusions on the characteristics of the race track, since all corners are assumed to have the same importance on the lap time. That's why it is necessary to make a second analysis using simulation.

Running simulations with varying grip factors allows us to understand the relative importance of each corner. By increasing the grip by five per cent in one corner at a time, we can understand which corners have the biggest influence on the lap time on a given race track.

Finally, we should also look at the energy spent by each tyre to better understand the work in each tyre. The goal is to make the tyre work in its ideal ranges of pressure, camber and temperature for its operation conditions. These conditions are defined by both the track layout (turn direction, cornering speed) and vehicle design/set-up parameters.

In this article we described another method to characterise a race track, and why you might want to run your car with an asymmetrical set-up. This and other methods are discussed in depth at the OptimumG Data Driven Performance Engineer Seminar. We explain, step by step, how to process the data, make interpretations, and draw valuable conclusions. To find out more about the seminars' content and dates, visit us at optimumg.com

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