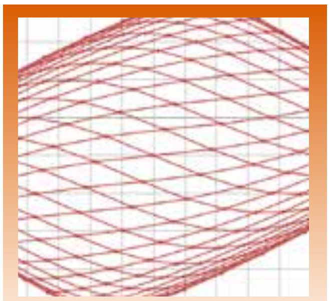


Slip Angle provides a summary of OptimumG's seminars

Slide rules: analysing an oversteering car

What makes a car quick in steady state and in transient? Claude Rouelle develops his analysis of lateral acceleration and yaw moment variation

Days of yaw: too much yaw moment at the apex and you have oversteer, too little and it's understeer. For drifters too much yaw moment is obviously a good thing

In April's *RE* (V27N4), we saw that there are 12 causes for the yaw moment: four tyre lateral forces *Fy*, four tyre longitudinal forces *Fx*; and four tyre self-alignment moments *Mz*. Let's now imagine a car cornering and braking in a left-hand corner entry, and consider this anti-clockwise yaw moment as positive.

The yaw moment equation is as follows: $(Fy_{LF} \cos \delta_{LF} + Fy_{RF} \cos \delta i_{RF})\, a - (Fy_{LR} + Fy_{RR})\, b + Fx_{LF}\, Tf/2 + Fx_{LR}\, Tr/2 - Fx_{RF}\, Tf/2 + Fx_{RR}\, Tr/2 - Mz_{LF} - Mz_{RF} - Mz_{LR} - Mz_{RR} = Izz\,(dr/dt)$ (7) **(Figure 1)**. In this case, the two front tyre lateral forces *Fy* as well as the two left side braking forces *Fx* create a positive yaw moment, while the two rear tyre lateral forces *Fy* and the two right side braking forces *Fx* create a negative yaw moment. Except for some exceptional cases of very high slip angles, tyre self-alignment moments *Mz* are most often negative. This equation is made in the chassis coordinate system, which is why the cosines of the inside and outside front steer angle are used. We will consider any possible static front and rear toes (we could call those 'pre-slip angles') and any possible bump steer and steer by compliance as negligible.

Electronic Stability Programming (ESP), front and/or rear differential control or torque vectoring (especially in the case of cars with four electrical motors) are often used to control the tyres *Fx* and, consequently, the yaw moment. However, it is important to notice that on practically all cars, the distances *a* and *b* (that are the leverages of the tyre lateral forces) are bigger than the front or rear half-track (that are the leverage of the tyre longitudinal forces). In the case of a tyre friction ellipse that is a circle (as much potential *Fx* as potential *Fy*), tyre braking or acceleration longitudinal forces will always have a smaller effect on the yaw moment than the tyre lateral forces.

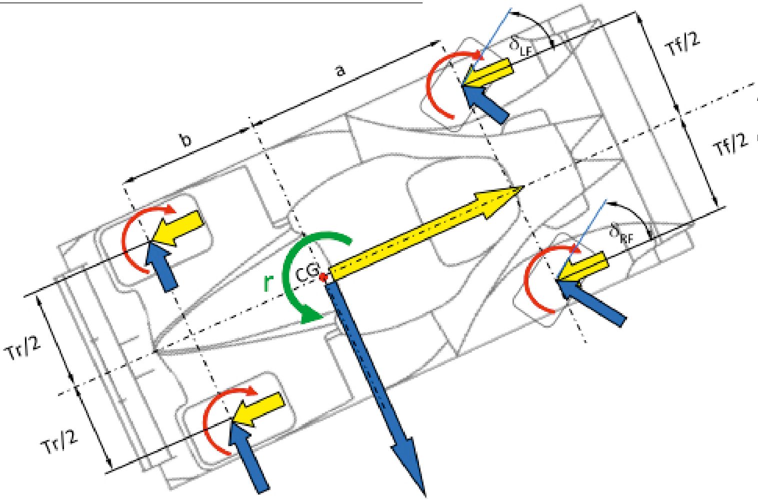
Lateral thinking

Several factors such as slip angle, dynamic vertical load, camber, speed, pressure, and temperature influence the tyre lateral forces. Tyre slip angles have three causes: car side slip angle *ß*, yaw velocity *r*, and steering angle *ß*. As *ß*, *r* and *ß* evolve in a turn so do the four slip angles, the tyre vertical loads, cambers, temperatures and lateral forces.

We will now analyse the evolution of both lateral acceleration and yaw moment along a turn, and to simplify our thoughts here, we will only consider the tyre lateral forces' influence on the yaw moment and the lateral acceleration.

Starting from point A seen on **Figure 2,** the driver turns the steering wheel and creates front tyres steering angle, always with some delay as even the best designed and manufactured cars have some steering compliance.

A reaction force to the front tyre lateral forces is created at the car's CG



$$(Fy_{LF}\cos\delta_{LF} + Fy_{RF}\cos\delta i_{RF})\,a - (Fy_{LR} + Fy_{RR})\,b + Fx_{LF}\,T_f/2 + Fx_{LR}\,T_r/2 - Fx_{RF}\,T_f/2 + Fx_{RR}\,T_r/2 - Mz_{LF} - Mz_{RF} - Mz_{LR} - Mz_{RR} = Izz\,(dr/dt) \quad (7)$$

Figure 1: Equation of the yaw moment with its 12 causes

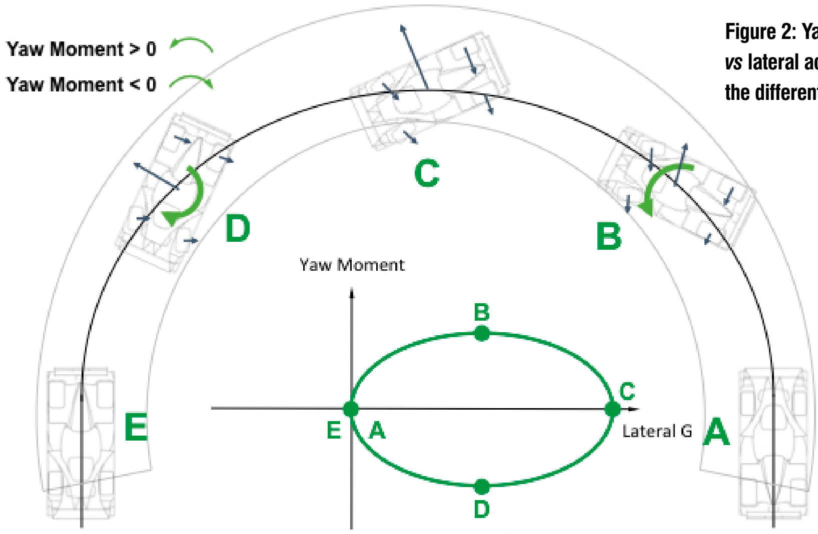
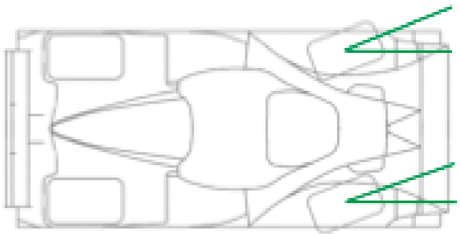
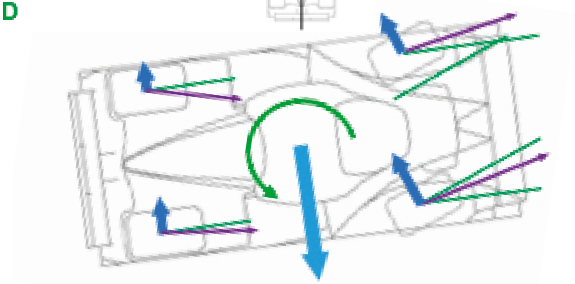


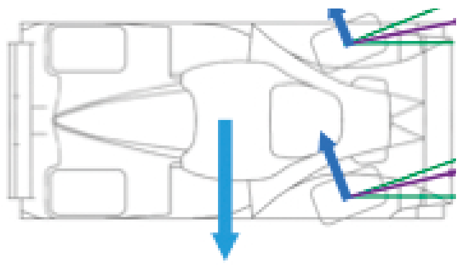
Figure 2: Yaw moment *vs* lateral acceleration in the different turn phases



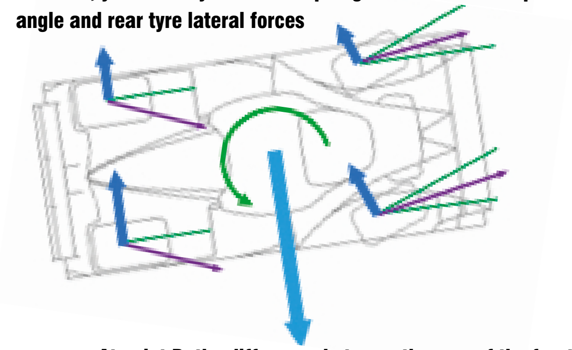
Point A: Steering wheel input



From point A to B: While front tyre lateral forces continue to increase, yaw velocity *r* and CG slip angle *ß* create rear slip angle and rear tyre lateral forces



Right after point A: Front slip angle, front tyre lateral forces, lateral acceleration, and yaw moment increase



At point B, the difference between the sum of the front tyres and the sum of the rear tyre lateral forces is the biggest: the yaw moment is at its peak. At point C (the corner apex), the front and rear lateral tyre forces will reach their peak and the yaw moment created by the front tyres will equal the yaw moment from the rear tyres. The yaw moment is 0

Figure 3 (above and right): The different phases of tyre lateral forces and yaw moment in the corner

Front tyre lateral forces are created, again with some delay that we often define with the tyre relaxation length. A reaction force to the front tyre lateral forces is created at the car CG. The front tyre lateral forces are not yet balanced by any rear tyre lateral forces, as for a very short time there isn't any rear slip angle. However, there is a yaw moment. Without it, the car wouldn't enter the corner. That yaw moment will make the car slip (angle *ß*) and yaw (yaw velocity *r*), and so are the rear slip angles and rear tyre forces being built, again considering some reaction time due to the rear compliance and tyre transient behaviour. The process will continue until point C where the yaw moment generated by the front tyres will be equal to the yaw moment generated by the rear tyres. The yaw moment is now 0.

Let's suppose for a moment that the car was driven on a huge surface without any obstacles. If at point C the yaw moment would remain 0, the car will put itself on a skid pad. We need to create a 'de-yawing' moment to extract the car from the corner. The driver will reduce the steering wheel angle. The front tyres steering angle, front slip angles and front lateral forces will be reduced earlier than the rear ones and a negative yaw moment will be created. The apex is the region of the corner where the yaw moment is 0. If you are not convinced, just look at the trace of your gyro signal. It will be flat for a short time, which means no yaw acceleration and no yaw moment. In a simplified way, we could look at the apex as the place where the racecar is close to the steady state definition. But this is also where the gyro slope and, therefore, the yaw moment sign change.

Figure 3 shows the evolution of both yaw moment and lateral acceleration in a left-hand corner.

Apex speed

Let's now suppose that an engineer wants to increase his car speed at the apex. If he could increase all corners'

Some engineers say there isn't such a thing as understeer or oversteer – for them there is only under or over yaw moment

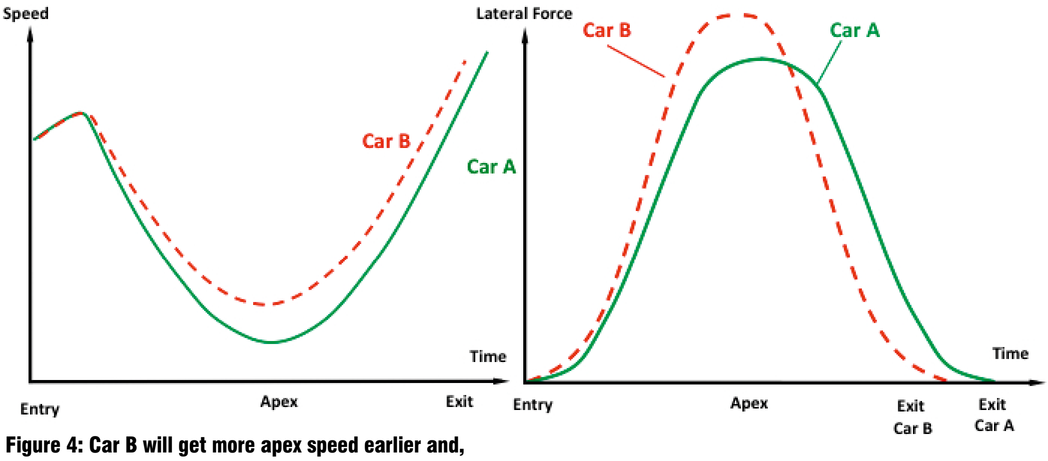


Figure 4: Car B will get more apex speed earlier and, therefore, also earlier and more lateral acceleration

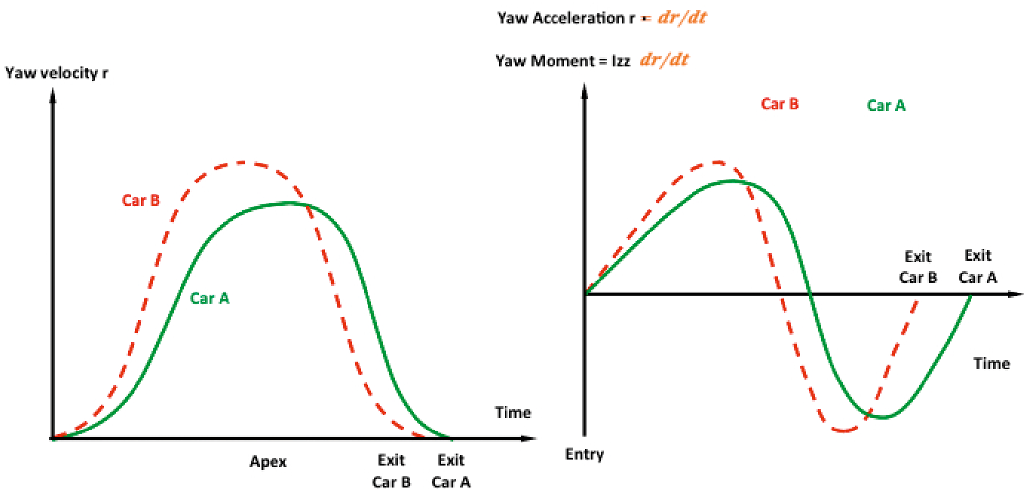


Figure 5: Car B will get more yaw velocity, more yaw acceleration, and more and earlier yaw moment

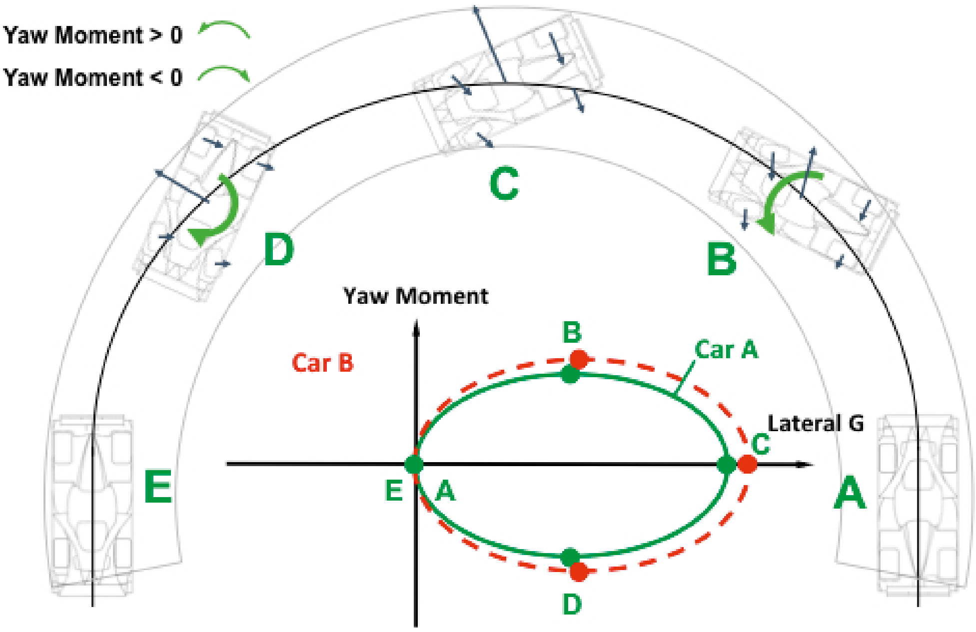


Figure 6: Going faster not only requires more tyre grip but also the right amount of yaw moment, which means the right difference between front and rear tyre grip (or left and right tyre grip)

apex speed by 0.5 per cent, the lap time gain would be significant. Let's imagine that the driver is on the same trajectory with car B that he was with car A so we can make an apple-to-apple comparison. More apex speed on the same given radius means more lateral acceleration at an earlier time and a necessary increase in tyre grip **(Figure 4).** There are many ways engineers could reach this goal: better tyres; the same tyres with more vertical load from aerodynamic downforce; or better use of existing tyres with appropriate slip angle, pressure, camber, temperature, etc.

That is for lateral acceleration. But what about the need of yaw moment and yaw moment variation? More tangential speed on the same radius implies more yaw velocity. A bigger yaw velocity in less time implies a bigger yaw acceleration. Bigger yaw acceleration for a given yaw inertia means bigger yaw moment **(Figure 5)**. The yaw acceleration (or yaw moment) *vs* lateral acceleration diagram will be different **(Figure 6)**.

Yaw the boss

Some engineers often say that there isn't such a thing as understeer or oversteer – for them there is only under or over yaw moment. Going faster is not only about getting more tyre grip, it is also about getting the right amount of yaw moment at the right place in the corner. Too much of it, and you have an oversteering car. Too little of it, and you have an understeering car.

Everybody understands that to gain more speed we need better tyres and/or better use of tyres. But going faster is also about getting the right amount of yaw moment and, consequently, the right amount of difference between front and rear tyre lateral forces (or left and right tyre longitudinal forces).

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